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Usage of Tinker Plots to Address and Remediate 6th Grade Students' Misconceptions about Mean and Median

Zuhal Yilmaz

Elementary Mathematics Education Department, Zirve University, Gaziantep, Turkey, 27260
E-mail: zuhal.yilmaz@zirve.edu.tr

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ABSTRACT Current need for interpreting data, making inferences from existing data, leads to an increased emphasis on the teaching of statistics in mathematics curricula. Recent studies suggested that using educational technology supports students' meaningful understanding of statistics. This study addresses the importance of technological tool usage to introduce introductory statistical concepts; mean and median and diagnose student's misconceptions about these concepts. Three teaching experiment sessions were conducted with three 6th grade US students and this study reported one of the students' work in the teaching experiment. Dynamic data exploration software TinkerPlots was used in the study. Study findings indicated that appropriate usage of the TinkerPlots support enhance students' meaningful understanding about mean and median. In addition the proper usage software gives a mean for teachers to diagnose and remediate students' statistical misconception.

INTRODUCTION

National Council of Teachers of Mathematics (NCTM) in *Principles and Standards for School Mathematics (PSSM)* indicated that "the existence, versatility, and power of technology make it possible and necessary to reexamine *what* mathematics students should learn as well as *how* they can best learn it" (NCTM 2000). In order to ensure better teaching and learning before implementing a task in the classroom teachers should consider possible learning obstacles, misconceptions that prevent students' learning, students' prerequisite knowledge and experience, possible strategies students may use while working on presented task (Stein and Smith 2012). In addition, teachers should address mathematics behind the task through meaningful classroom discourse (Stein and Smith 2012). This paper will address the role of technological tool, TinkerPlots (a dynamic data exploration software), in helping teachers to introduce introductory statistical concepts of mean and median to students and to diagnose students' misconceptions around those concepts.

Address for correspondence:
PhD. Candidate Zuhal Yilmaz, PhD.
Student at Middle East Technical University,
Elementary Mathematics Education Program,
Research Assistant at Zirve University,
Zirve University Kitil Hisar Kampusu Sahinbey,
Gaziantep, Turkey, 27260,
E-mail: zuhal.yilmaz@zirve.edu.tr ,
zyilmazncsu@gmail.com

Theoretical Background

Guidelines for Assessment and Instruction in Statistics Education (GAISE) curriculum framework for PreK-12 stated as a result of data richness during information age coupled with rapid changes in technology and modern methods in data analysis during 1980s, developing curriculum materials started for introducing statistical concepts as early as elementary grades. (Franklin and Garfield 2006; Chance et al. 2007).

Nowadays, several researchers support the idea that students should actively participate in the learning process. In this process, students construct their own knowledge through restructuring their existing knowledge (Cobb 1994). This view of learning progress described in constructivist theory in which learners seen as active builder of the knowledge and learners' experiments, environment and existing beliefs are key factor in the internalization of the newly encountered knowledge (Cobb 1994). According to Von Glasersfeld (1987), in constructivism information is not directly transmitted to students through procedural manner. In this transition learner has an active role in which learner seek for reasons not only memorize the facts. Teacher acts as a facilitator in the learning process. He/she responsible to create active learning environment and provide students with high level engaging tasks (Eggen and Kauchak 2001). In order to design this learning environment teachers need certain skills to support students' mathematical and statistical learning and understanding. Those skills can be listed as 1) determining students initial knowledge level, 2) understanding how students thinking evolve over time, 3) launching mathematical tasks that elicit important mathematical strategies (Stein and Smith 1998; 2012) ideas and 4) eliciting possible learning obstacles and misconception of students (Confrey 2006), and 5) actively refining their academic content and pedagogical content knowledge (Graeber et al. 1989).

A Brief History of Statistics Teaching

In the past statistics teaching and learning mainly focused on basic computational skills and procedures (Ben-Zvi 2000). In parallel to constructivist view of learning, after a while researchers noticed an important drawback of this approach in which students had difficulties with explaining a simple phenomena or a relation (Konold and Kazak 2008). For instance, students could not compare the averages of two distributions (Hancock et al. 1992; Watson and Moritz 1999). As a result, researchers searched ways for making statistics more meaningful and accessible to students. The major move was increasing usage of technology in statistics education in line with exploratory data analysis (Tukey 1977). In this approach students seek for describing, analyzing, organizing and representing data. Ben-Zvi (2000) stated that technology is best-fit tool to provide these opportunities to students as they engage the data analysis process. Biehler (1998) indicated that integration of technology in teaching might enhance students' learning of statistics. Since usage of statistical software provides students with opportunities in which they can interpret, organize, represent and analyze data in various rich ways. These first hand learning experiences support students' to construct their own understanding about related statistical concepts.

TinkerPlots and Misconceptions Related to Measures of Central Tendency

TinkerPlots is one of the statistical software designed for fostering statistical thinking and understanding. Konold and Miller (2005) designed this tool for helping middle school students to collect, organize, analyze, represent and interpret data in relation to real life context for developing their statistical thinking. With the usage of this type of statistical software data analysis is not accessible just only statisticians, it is now accessible to students and teachers (Batenero et al. 2008). They can engage projects that require statistical analysis and reasoning (Batenero et al. 2008).

According to Konold and Miller (2005) TinkerPlots allows students systematically to build their understanding of statistical representations and concepts through exploring data. TinkerPlots has the tools serve for students to test their hypothesis, to construct different statistical representations, to see how different data affects their analysis and to see in what degree they need additional data to learn more about nature of the given real life phenomenon (Rubin 2007). Also, Rubin (2007) stated that usage of these tools lead multiple data representations for fruitful classroom discussions and it promotes inquiry based learning in classroom settings. One other important way to facilitate students learning of statistical concepts is to have an understanding of how students perceive these concepts. At this point, it is important to detect students' conceptions and misconceptions. The major students' misconceptions (Li and Shen 1992; Batenero et al. 2008) about measures of central tendency revealed in the researches are:

1. Students made errors while finding two weighted means
2. Li and Shen (1992) shows that many students forget to deal with frequency of the given data when they are computing the mean
3. Students know how to calculate the mean and assume this calculation satisfies these axioms, identity element and associativity. Students assume that score of 0 do not affect overall scores mean of an examination scores
4. Students define mean as midpoint or most frequent value

All these misconceptions pointed " knowledge of a computational rule not only does not imply any deep understanding of the underlying concepts, but may actually inhibit the acquisition of a more complete conceptual knowledge" (Batenero et al. 2008).

Although TinkerPlots is a strong tool for fostering statistical reasoning, fostering statistical thinking depends on how teachers use this tool with their students. The usage of these types of software in classroom also has some pedagogical impacts. Usage of tools like TinkerPlots in classroom also affects the way teachers engages with statistical reasoning, the way teacher teach statistics. Lee et al. (2010) pointed the importance of teachers' pedagogical expertise as they use these tools while engaging students' with data analysis. They stated that teachers should know how to use these tools in their classroom as they teach statistics. Garfield and Ben-Zvi (2009) stated without training teachers on how to use these technological tools in their classrooms activities, these resources would not have impact on students' learning. Moreover they stated in order to create a meaningful learning experience for students to foster statistical thinking teachers should know how effectively engage with these tools, should develop statistical reasoning.

This study reports on how a teacher uses TinkerPlots to promote student's conceptual understanding about mean and median. Zan was the most expressive student among the students that participated in the study. She also showed clear misconceptions that students might encounter while engaging the concepts of mean and median. Also, her case was an excellent case that shows how usage of technology supports students' statistical understanding through firsthand experience. As a result, this study will reports on Zan's case how she perceived statistical concepts mean and median, how usage of TinkerPlots helped her to revise her initial understanding about these concepts.

METHODOLOGY

In this study, teaching experiments methodology used to identify and understand students' understanding about the statistical concepts: mean and median and to identify the benefits of usage of TinkerPLots while identifying students' conceptions. Moreover, according to Steffe and Thompson (2000) attributions about students' mathematical understanding and operations attached two constraints. Those are "…the language and actions they are able to bring forth in students [and] … students' mistakes, especially those mistakes that are essential; that is, mistakes that persist despite researchers' best efforts to eliminate them"(p. 267). According to him students' current mathematical knowledge included the sources of those essential mistakes. Teaching experiments is one of the effective ways for researchers to experience those constraints (Steffe and Thompson 2000).

In this study three session of teaching experiments, each lasts 50 minutes, were conducted with three 6^{th} grade students in United States. Conducting teaching experiments with three children, instead of a whole classroom teaching experiment, informed the study in terms of deep analysis of each student's works in terms of their misconceptions and statistical reasoning. The limitation in the number of the students helped to track each student's works during each experiment and acts on the computer screen. Also, this number limitation facilitated ongoing interaction between researcher and student during each session. In the teaching experiment, real world mathematical tasks and a technology integrated mathematics lesson layout was used. In each session of teaching experiment, TinkerPlots was used as a technological tool. Before the teaching experiments all three students attended a workshop about how they use this software. The task and teaching experiment layouts were designed to elicit students' understanding about mean and median. Students' work from teaching experiment was analyzed by using qualitative analysis.

In the first session of the teaching experiment, initially, students shared their initial knowledge about mean and median concepts. Then they collected real world data to analyze them. In the second session, students tried to make sense of the given situations through firsthand experience by using TinkerPlots. As NCTM learning standards states that, "Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (NCTM 2009). Usually many students have some idea about mean as in the sense of average from their daily life experiences. Although these prior ideas are not necessarily always correct they have a strong influence on students' conception about mean, median. Some of those initial experiences of students' may become a learning obstacle for them.

In the second session of the teaching experiment, students formulated their conjectures about the mean and median of their collected data (for example, height of the boys and girls in their classroom). And they tested which factors affected the mean and median of their data set.

In the third session of the teaching experiment, students tested their conjectures and then revised them if needed. They stated their final understanding about mean and median concepts and summarized what they have learned.

Through recording each teaching experiment, a detailed description of students' work, thought processed and misconceptions were captured while students' working on the presented task. According to Powell et al. (2003), video data allows researcher to play with the recorded data multiple times in different format: slow and fast motion, frame by frame, forward and backward reply. In this study this capability of video data were utilized at different times with other colleagues so that detailed aspects' of students' work could be captured. This multiple viewing of data with other colleagues at different times utilized as a way to ensure triangulation so that reliability of the study (Powell et al. 2003).

In this study Powell et al. (2003) suggested analytical model were use to analyze the video data as follows: "1) Identifying critical events, 2) transcribing [related sections and constructing video clips], 3) coding, 4) constructing storyline, and 5) composing narrative" (p. 413). In the first part two critical events were determined as follows:

1. A clear misconception about the mean and median concept were aroused while students working on the tasks
2. Students' solution strategies and thought processes changes after the interaction with TinkerPlots

Then video clips were constructed based on the identified critical events. Then coding schema of the study composed through multiple viewing of those video data and shared with other educators and researchers. This helped me to clearly formulate my coding schema. Four coding were identified as follows: 1) Calculation of mean and median without having a deep mathematical understanding 2) Clearly stated/observed misconceptions (related to weighted means, identity element (effect of adding zero into the data set) and definition of median 3) Revisions were made by students' to their misconceptions and initial hypothesis after interacting with TinkerPlots. In the last part related transcribed sessions used to compose narratives as a empirical evidence for reporting purposes in the findings part.

RESULTS

In this part, student's statistical understanding will be analyzed in three process cycles as, student's initial level of understanding of mean and median concepts, changes in student's understanding during the teaching experiment and student's level of understanding of these concepts at the end of whole process.

Analysis around Mean Concept

Watson and Moritz (2000) stated that students might have an understanding of calculation procedure of statistical measures mean, median and mode without having deeply understanding their mathematical properties. In Zan's case, firstly she defined mean of the height data set as "It is the middle point of my data." After that, researcher asked Zan "Can you check your assumption by using TinkerPlots to see it is always middle point of your data?" Then she checked her assumption with two numbers first as 140 and 160 and see that the mean is half of the sum of two data measures (Fig. 1).

In the previous case, the mean was still the middle value of the given data set. So, the researcher asked "if you have one more height data measure, what happens to your mean?" Then, she added one more data measure and realized that the mean was not the middle measure of her data set. Usage of technology, Tink-

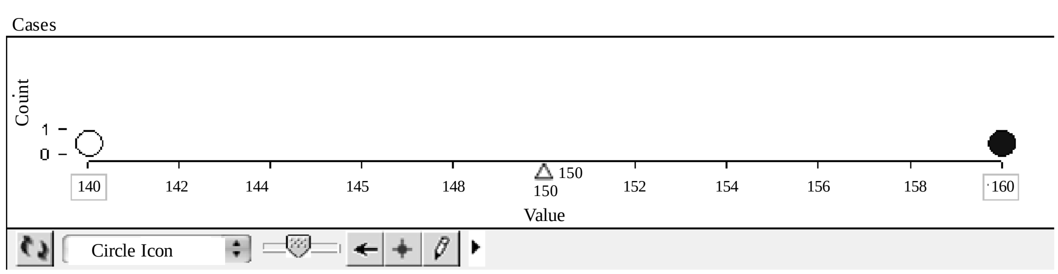

Fig. 1. Student's work: Mean of 140cm and 160cm is 150cm

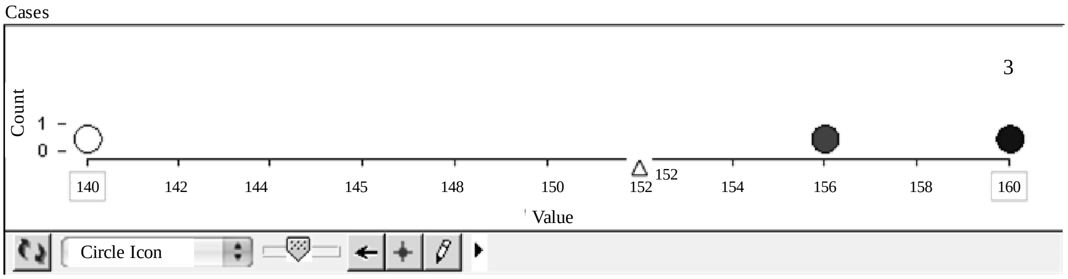

Fig. 2. Representation of adding a greater data measure (156cm) than the previous mean (150cm), the new mean is 152cm

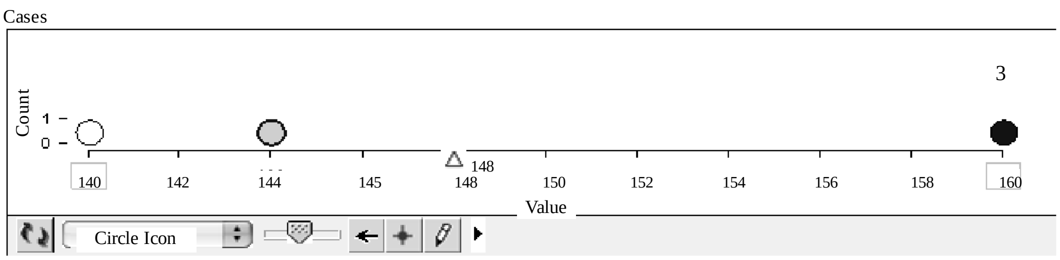

Fig. 3. Representation of adding a smaller data measure than the first mean (150cm). The new mean is 148cm

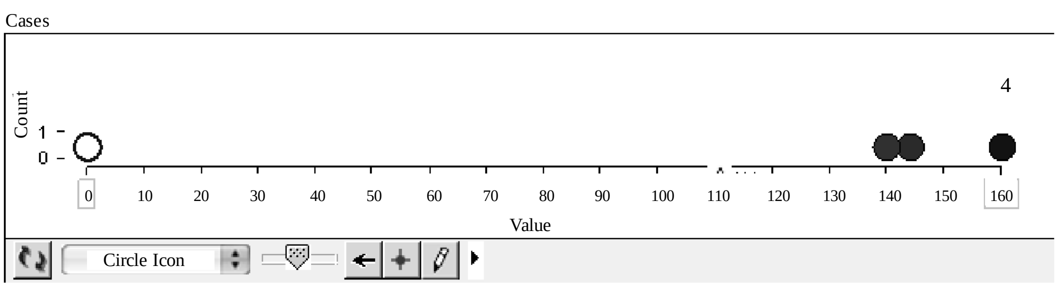

Fig. 4. Representation of adding a smaller data measure (zero) to the existing three measures. The new mean is 111cm

erPlots, created contradiction with her initial understanding of mean: the middle point of her data set. TinkerPlots assisted her to attach the meaning of the definition of mean into her experience. She continued to check her assumption by adding or removing data from her existing data set in Tinker Plot and recorded her findings about the mean and median of each data set in her data table (Figs. 2 and 3). Eventually she defined the mean as, “sum the all data measures and divide the total number of data.” As, Ben-Zvi (2000: 128) pointed that, usage of technology leads active construction of knowledge related to mean by “doing” and “seeing” statistics.

The next part of the teaching experiment continued with addressing Zan’s interpretation of frequency of the data concept while calculating the weighted means. In order to assess her initial perception, a word problem was posed as, “There are 10 students in the classroom 6 of them boys and 4 of them girls. The average height of girls is 155cm and the average height of boys is 170cm. What is the average height of ten students in this class?” She said, “since we have two groups: boys and girls, I add two means and divide by number of groups: two.” After that, the researcher let her to keep continue on the given activity sheet, since in the activity sheet student asked for finding mean of the data measures by adding additional height measures, which were sometimes equal measures, sometimes not. Also, during the process the researcher asked "If you have two equal height measure data and one different height data measure, how do you find you mean?" She tried several scenarios in TinkerPlots and observed the mean behavior in each case. Eventually, she reached four conclusions:

1. "If I have some data measures and I add some new measures into my data then my mean will affect."
2. "If I add a data greater than my existing mean, the new mean will increase." Figure 2 shows one of the student's works.
3. "If I add a data smaller than my existing mean, the new mean will decrease." Figure 3 shows one of the student's works.

In this case, Zan understood that adding zero to her existing data set decrease the mean. Figure 4 represents this situation.

Usage of TinkerPlots helped to remediate the previously stated misconception: students know how to calculate the mean and assume this calculation satisfies these axioms, identity element.

4. "If I have equal measures of data, all my data measures contribute to my mean so, I need to divide by number of my data measures not number of groups."

Figure 5A showed that she added a data measure (152cm) into her existing data set and observed that mean changed. Figure 5B showed that she added the same data measure again and observed that the frequency of the data measure had an effect on the mean. Then she concluded : " in the first problem, I divided group's mean value sum by the number of the groups. Now, I understand that it is not correct. I need to divide my total measure by the number of the total people." First, she multiplied the number of the boys in the class with their avareage height measure of boys to find the total height measures of boys in the class. Second, she multiplied the number of the girls in the class with their avareage height measure of girls to find the total height measures of girls in the class. Last, she added these two total height measures of boys and girls. Then, she divided the total measure by the total number of the students in the class to find the average of height measure of the class. Figure 6 represents Zan's solution process for the first word problem.

The dynamic nature of TinkerPlots allowed Zan "to focus on interpretation of results and understanding concepts rather than on computational mechanics" (Chance et al. 2007). Zan had chance to add, remove and drag cases and reflected observed phenomena and revised her initial thoughts about how to calculate weighted means and how to deal with frequency of data measures while calculating the weighted mean.

Analysis Around Median Concept

Zan's initial conception about median was: "it is the middle of the scale". She showed what it means to be the middle of the scale by adding the smallest and biggest value from the data set and divided by two. Then she described the final value: "this is my median." After that, the researcher asked to Zan "Can you check your assumption by using TinkerPlots?" With the help of the activity sheet, she tried several scenarios and note down the changes in her median. In her first trial in TinkerPlots, she added two data point to the graph and noted on the worksheet

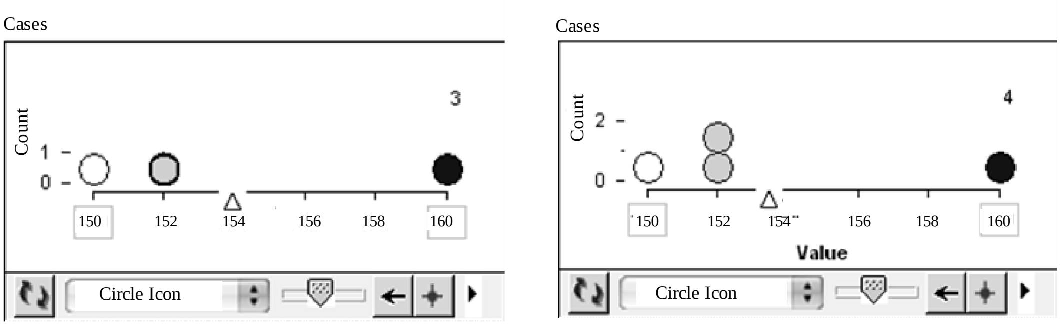

Fig. 5A/5B: Zan's work: Examining the effect of frequeny of data measures on weighted means

that her conjecture held. Then she tried three data measures and noted that, her conjecture did not hold. At that point, the researcher posed the question "why your conjecture did not hold?" She said "in this case it is the mid value of my data set, not the value for adding biggest and lowest number and divided by two." She added one more data measure to the existing three data measures. She said "My median is changed again."

Based on overall observation that she made, she realized that, number of the data measures in the data set affect her median and she revised her conjecture. Her new conjecture was: "My median cannot be half of the sum of biggest and smallest number in my data, it should be something else." Then, the researcher asked, "What it means to be something else? What sources may help you to figure this out?" Then she turned back to her notes about each trial that she made. She tried to catch up a pattern. At the end of the teaching experiment, she realized that, when she had an odd number of data measures the median would be the middle data measure of the set. Also, when she had an even number of data measures the median would be half of the sum of two middle data measures.

DISCUSSION

Designing teaching experiments around revealing misconceptions and determining prior knowledge is parallel with NCTM's teaching principle: "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well." Technology usage has important role for teaching mathematics effectively. In this study, proper usage of TinkerPlots software challenges students to organize data, order data and explore the mean, median and mode by using real life cases. Table 1 represents benefits of usage of technology (Ben-Zvi 2000) in statistics education and some related implications of three teaching experiments.

The important aspect of TinkerPlots is: it lets students' visualization of abstract statistical concepts and immediately reflects the changes in data set on the graph. TinkerPlots gives opportunity to students to apply what if case scenarios by adding or dragging data icons so students can observe and analyze what happens to mean, median and mode of the given data set. By doing so, they can realize which factors affect the mean, median and mode.

CONCLUSION

The discussed features of TinkerPlots a long with the meaningful tasks function as a tool for teachers to identify and remediate students' misconceptions related to mean and median concepts. In addition dynamic nature of the software help students to test their initial conjectures related to the factors that may affect mean and median of the given data. This testing process provides students with the opportunity to move beyond only procedural understanding of mean and median. This study supports the notion that, appropriate usage of technology lets students exercise statistics with real data that allow them to make sense the statistical concepts and their interpretations. Through active experience while engaging with tasks involved

Table 1: Benefits of technology usage and exemplars from teaching experiments with TinkerPlots

Technology usage benefits	*Example learning and teaching implications from teaching experiment*
Students' active construction of knowledge, by "doing" and "seeing" statistics	Students can use the collected data to form graphical representations in TinkerPlotStudents can form different data that has same mean, median and mode
Opportunities for students to reflect on observed phenomena	By adding and removing cases in the TinkerPlot and observe their effects on mean, median and mode For instance, they add identity element 0 cm to the cases so they will see identity element also effects mean, median so they can see there is no identity element for mean and median
The development of students' meta-cognitive capabilities, which emphasizes awareness about own learning and thought processes	Follow up questions after dealing with technology help students to be aware of their own learning and understanding. For instance, what do you notice when you remove a bigger value case from your data collection? What will happen if you remove the smallest value?

the statistical concepts mean and median, students have chance to observe the factors that affect the mean and median. Based on their observations students revise and refine their initial conjectures or misconceptions. In addition, TinkerPlots make the abstract changes in the mean and median accessible the students through concrete graphical display. Yet, the technology itself is not enough to create these opportunities and experiences. There need to be a teacher assistance to promote questioning, discussion in order to create a better understanding.

RECOMMENDATIONS

This study involves empirical evidences and ways for both teachers how they can integrate a particular technological tool, TinkerPlots in their current statistics teaching practices on particular concepts: mean and median. Further empirical studies should be carried out on different statistical concepts to frame the ways in which can a technological tool may assist statistics learning.

For the practitioner teachers this study reported some empirical evidences that suggests; integrating technology can enhance students' learning if teachers create meaningful and engaging activities that fosters their students' statistical reasoning. Since, through visual representations of the data and active manipulation of the data has potential to support students' understanding of abstract statistical concepts. As a result, this study suggests that teachers has opportunity to utilize these tools in their teaching practices that has potential to enhance and facilitate their students' statistical learning.

NOTE

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